Étale cohomology of curves

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Lemma 0.1. *Let X be a finite scheme over a separably closed field k. Then $\mathrm{H}^i(X, \mathcal{F}) = 0$ for $i > 0$ and for all $\mathcal{F} \in X_t$.*

Proof. in progress... □

Lemma 0.2. *Let X be a scheme, $i : Z \to X$ a closed immersion, and $\mathcal{F} \in X_t$ supported on Z. Then the canonical morphism $\mathcal{F} \to i_* i^* \mathcal{F}$ is an isomorphism.*

Proof. in progress... □

Corollary 0.1 (of Lemma 0.2)**.** *Let X be a scheme and $\mathcal{F} \in X_t$ supported on finitely many closed points of X. Then*

$$\mathcal{F} \cong \bigoplus_{x \in \mathrm{supp}(\mathcal{F})} x_* x^* \mathcal{F}$$

Proof. in progress... □

Lemma 0.3. *Let X be a scheme and $\mathcal{F} \in X_t$ skyscrapersheaf $f : x \to X$ (riscriverò meglio). Then*

$$\mathrm{R}^i f_* \mathcal{F} = 0$$

In particular,

$$\mathrm{H}^i(X, \mathcal{F}) = 0$$

for all $i > 0$.

Proof. in progress... □

Lemma 0.4. *Let X be a quasi-compact quasi-separated (qcqs) scheme, and $\mathcal{F} \in X_t$ be a torsion sheaf. Then $\mathrm{H}^i(X, \mathcal{F})$ is a torsion group for all $i \geq 0$.*

Proof. in progress... □

Theorem 0.1. *Let C be an algebraic curve over a separably closed field k. Then*

$$\mathrm{H}^i(C, \mathcal{O}_{C,t}) = \begin{cases} \mathcal{O}_C(C)^\times & i = 0 \\ \mathrm{Pic}(C) & i = 1 \\ 0 & i > 1 \textit{ and } k \textit{ is algebraically closed} \\ p^\infty \textit{ torsion group} & i > 1 \textit{ and } k \textit{ is non perfect with char(k)=p} \end{cases}$$

Proof. in progress... □